Robust Distributed Detection in Massive MIMO Wireless Sensor Networks under CSI Uncertainty

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***Abstract*—This paper presents a Neyman-Pearson (NP) criterion based optimal distributed detection framework for a massive multiple-input multiple-output (MIMO) wireless sensor network (WSN). Robust fusion rules are determined for the local decisions transmitted by the sensor nodes, considering the availability of both perfect as well as imperfect channel state information (CSI) at the fusion center. Further, the probability of error of the individual sensor decisions, which arises in practical scenarios, is also incorporated in the decision framework. Closed form expressions are derived to characterize the resulting probabilities of detection and false alarm for the system. Simulation results are presented to demonstrate the improved performance of the proposed detectors in comparison to the existing detectors and to validate the theoretical findings.**

I. Introduction

Massive MIMO communication has recently attracted significant interest for potential application in wireless sensor networks (WSNs). This is motivated by the fact that the large antenna array employed at the base station/ fusion center can simultaneously serve several sensors [1], in turn leading to various benefits such as increased spectral efficiency, reduced latency, simplified signal processing [2], improved reliability and power efficiency [3].

The distributed detection problem, where multiple sensors send their individual test statistics to be combined at the fusion center, has been widely studied in the existing literature [4], [5]. The work in [6] considers fusion of analog sensor observations, whereas the work in [5] combines the local sensor decisions. On the other hand, the scheme in [7] proposes decoding followed by fusion of the individual sensor test statistics in order to make a global decision. Similarly, the works in [8], [9] consider a MIMO system model to develop detectors that are robust to the uncertainty in the CSI statistics. Moreover, in [9], the authors develop a deflection coefficient based optimization framework to obtain optimal signaling vectors, which can further enhance detection performance. However, all the above works consider an orthogonal multiple access channel (MAC) between the sensors and the fusion center. Employing a coherent MAC-based WSN leads to a significant reduction in the communication bandwidth. Based on this, the authors in [10] develop the likelihood ratio rest (LRT) based Max-Log, modified maximal ratio combiner (MMRC) detectors for a large antenna array MIMO WSN. However, the detectors presented in [10] consider identical local detection performance, i.e., all the sensors have identical probabilities of detection and false alarm, which restricts the applicability of the framework proposed therein.

This work considers a coherent MAC-based WSN comprising of a massive antenna array based fusion center with multiple sensor nodes and significantly extends the results in [10]. Each sensor is assumed to transmit a signal vector that conveys its local decision to the fusion center. NP criterion-based novel detection rules are determined for the distributed detection scenario described above in a massive MIMO WSN, incorporating also non-idealities such as the probabilities of detection $(P_{D,k})$ and false alarm $(P_{F,k})$ of the local sensor decisions. Further, the proposed framework considers the availability of both perfect and imperfect CSI at the fusion center, thus rendering the resulting decision rules robust. Closed form expressions are obtained for the system level probabilities of detection and false alarm at the fusion center to analytically characterize the performance of the proposed detectors. Finally, simulation results are included to demonstrate the improved detection performance of the proposed schemes.

II. System Model

Consider a wireless sensor network (WSN) wherein the sensors are sensing a specific signal of interest. This scenario can be modeled as a decentralized binary hypothesis testing problem, where hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ correspond to the absence and presence of the target of interest, respectively. The WSN consists of K sensors, each having a single antenna, transmitting to the fusion center equipped with a large antenna array of $M \gg K$ antennas over a flat fading multiple access channel (MAC). The kth sensor, $1 \leq k \leq K$, transmits a vector $\mathbf{x}_k \in \mathbb{C}^{L\times 1}$ over L time instants and can take values $\mathbf{x}_k = \mathbf{u}_k$ or $-\mathbf{u}_k$ corresponding to its local decisions for the presence or absence of the target of interest, respectively. The probabilities of detection and false alarm for the kth sensor, denoted by $P_{D,k}$ and $P_{F,k}$, respectively, are characterized as

$$\begin{aligned} P_{D,k} &= \Pr(\mathbf{x}_k = \mathbf{u}_k|\mathcal{H}_1), \\ P_{F,k} &= \Pr(\mathbf{x}_k = \mathbf{u}_k|\mathcal{H}_0). \end{aligned} \tag{1}$$

Therefore, the received signal $\mathbf{y}(l) \in \mathbb{C}^{M\times 1}$ at the fusion center corresponding to the composite transmitted signal vector at the lth instant which consists of symbols transmitted by all the K sensors, i.e. $\mathbf{x}(l) = [x_1(l), \ldots, x_K(l)]^T \in \mathbb{C}^{K\times 1}$, $1 \leq l \leq L$, such that $x_k(l) \in \{u_k(l), -u_k(l)\}$, can be expressed as

$$\mathbf{y}(l) = \sqrt{p_u}\mathbf{G}\mathbf{x}(l) + \mathbf{n}(l), \tag{2}$$

where $\mathbf{G} \in \mathbb{C}^{M\times K}$ denotes the channel matrix between the fusion center and the K sensors, such that $g_{mk} = [\mathbf{G}]_{mk}$ represents the channel coefficient between the mth, $1 \leq m \leq M$, antenna at the fusion center and the kth sensor, p_u is the average transmit power of each sensor and $\mathbf{n}(l) \in \mathbb{C}^{M\times 1}$ is the circularly symmetric additive white Gaussian noise which can be modeled as $\mathbf{n}(l) \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2\mathbf{I}_M)$. The channel coefficient, g_{mk} is represented as $g_{mk} = h_{mk}\sqrt{\beta_k}$, where h_{mk} denotes the small-scale fading coefficient, and $\sqrt{\beta}_k$ accounts for the pathloss and log-normal shadowing that remains constant over multiple coherence intervals and is independent across m. Moreover, the channel coefficients h_{mk} are assumed to be independent and identically distributed (i.i.d.) symmetric complex Gaussian random variables, i.e., $h_{mk} \sim \mathcal{CN}(0,1)$. Hence, the channel matrix is given as

$$\mathbf{G} = \mathbf{H}\mathbf{D}^{1/2}, \tag{3}$$

where $\mathbf{H}$ denotes the small-scale fading matrix and $\mathbf{D}$ represents a diagonal matrix with the large-scale fading coefficients β_k, $1 \le k \le K$, along its principal diagonal, i.e. $[\mathbf{D}]_{kk} = \beta_k$. Using (2), the received signal at the fusion center obtained by concatenating the L transmitted signal vectors $\mathbf{x}(l)$, i.e. $\mathbf{X} = [\mathbf{x}(1), \ldots, \mathbf{x}(L)] \in \mathbb{C}^{K\times L}$, can be expressed as

$$\mathbf{Y} = \sqrt{p_u}\mathbf{G}\mathbf{X} + \mathbf{N}, \tag{4}$$

where $\mathbf{Y} = [\mathbf{y}(1), \ldots, \mathbf{y}(L)] \in \mathbb{C}^{M\times L}$ is the received signal matrix, $\mathbf{N}$ denotes the noise matrix obtained by stacking the vectors $\mathbf{n}(l)$, and elements of $\mathbf{N}$ are i.i.d. random variables, i.e. $n_{i,j}(l) \sim \mathcal{CN}(0, \sigma_n^2)$. The elements of $\mathbf{g}_k$ are zero-mean i.i.d. random variables, under favorable propagation conditions [11], with variances $\mathbb{E}\{|g_{mk}|^2\} = \beta_k$. Further, the different column vectors of matrix $\mathbf{G}$ corresponding to different sensors, are assumed to be mutually independent. This assumption holds in a multi-sensor massive MIMO WSN since the sensors are typically spatially separated by large distances. Therefore, by the law of large numbers, it follows that

$$(1/M)\mathbf{G}^H\mathbf{G} \approx \mathbf{D}, \quad \text{for } M \gg K. \tag{5}$$

Further, as a result of the large antenna array, the random channel vectors corresponding to different sensors become pairwise orthogonal under the favorable propagation conditions described above.

III. Optimal Fusion rule with perfect CSI

In this section, the optimal fusion rule is derived for the scenario with perfect CSI at the fusion center. The log likelihood ratio test (LLRT) based on the Neyman-Pearson (NP) criterion for the massive MIMO WSN system model in (4) is determined as

$$T(\mathbf{Y}) = \ln\left[\frac{p(\mathbf{Y}|\mathcal{H}_1)}{p(\mathbf{Y}|\mathcal{H}_0)}\right] \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma, \tag{6}$$

where $p(\mathbf{Y}|\mathcal{H}_1)$, $p(\mathbf{Y}|\mathcal{H}_0)$ denote the PDFs of $\mathbf{Y}$ corresponding to the alternative and null hypotheses and γ is the detection threshold. The expression for the LLRT in (6), can be determined by substituting the relevant quantities as,

$$T(\mathbf{Y}) = \sum_{l=1}^{L} \ln\left[\frac{\sum_{\mathbf{x}(l)} \exp\left(-\frac{\|\mathbf{y}(l)-\sqrt{p_u}\mathbf{G}\mathbf{x}(l)\|^2}{\sigma_n^2}\right)\Pr(\mathbf{x}(l)|\mathcal{H}_1)}{\sum_{\mathbf{x}(l)} \exp\left(-\frac{\|\mathbf{y}(l)-\sqrt{p_u}\mathbf{G}\mathbf{x}(l)\|^2}{\sigma_n^2}\right)\Pr(\mathbf{x}(l)|\mathcal{H}_0)}\right].$$

However, the LLRT presented above is computationally complex and numerically unstable due to the presence of exponential terms with large negative exponents [10]. Therefore, for a practically amenable implementation, a simplified LRT can be obtained by following the two step procedure described next. In the first step, the output is linearly filtered to recover the soft decisions of the individual sensors. In the subsequent step, a final decision is obtained incorporating the individual sensor decisions. The equivalent system model obtained after matched filtering in (4) is expressed as

$$\mathbf{Z} = \mathbf{G}^H\mathbf{Y} = \sqrt{p_u}\mathbf{G}^H\mathbf{G}\mathbf{X} + \mathbf{G}^H\mathbf{N}, \tag{7}$$

where $\mathbf{Z} = [\mathbf{z}(1), \ldots, \mathbf{z}(L)] \in \mathbb{C}^{K\times L}$ is the filter output matrix. For a given vector $\mathbf{x}(l)$, the filter output vector $\mathbf{z}(l)$ follows the complex Gaussian distribution $\mathcal{CN}(\sqrt{p_u}\mathbf{G}^H\mathbf{G}\mathbf{x}(l), \sigma_n^2\mathbf{G}^H\mathbf{G})$. Simplifying the expression in (7), using the result described in (5), the output of the matched filter at the lth time instant is $\mathbf{z}(l) = [z_1(l), \ldots, z_K(l)]^T \in \mathbb{C}^{K\times 1}$, $1 \le l \le L$, which can be determined as

$$\mathbf{z}(l) = \sqrt{p_u}M\mathbf{D}\mathbf{x}(l) + \tilde{\mathbf{n}}(l). \tag{8}$$

The quantity $\tilde{\mathbf{n}}(l) \in \mathbb{C}^{K\times 1}$ is a column vector of the noise matrix $\tilde{\mathbf{N}} = \mathbf{G}^H\mathbf{N} \in \mathbb{C}^{K\times L}$ with zero mean and covariance matrix $M\mathbf{D}\sigma_n^2$, i.e., $\tilde{\mathbf{n}}(l) \sim \mathcal{CN}(\mathbf{0}, M\mathbf{D}\sigma_n^2)$. Thus, with the aid of linear processing at the fusion center with a very large antenna array, i.e., $M \gg K$, the multiple access channel (MAC) between the sensors and the fusion center reduces to the parallel access channel (PAC) model above. The filter output vector for the kth user, corresponding to the vector $\mathbf{x}_k = [x_k(1), \ldots, x_k(L)]^T \in \mathbb{C}^{L\times 1}$, can be equivalently written as

$$\mathbf{z}_k = \sqrt{p_u}M\beta_k\mathbf{x}_k + \tilde{\mathbf{n}}_k, \tag{9}$$

where $\mathbf{z}_k = [z_k(1), \ldots, z_k(L)]^T \in \mathbb{C}^{L\times 1}$ and $z_k(l)$ follows the complex Normal distribution, i.e. $z_k(l) \sim \mathcal{CN}(\sqrt{p_u}M\beta_k x_k(l), M\beta_k\sigma_n^2)$. Hence, the LLRT based optimal test statistic $T(\mathbf{Z})$ for distributed detection in the massive MIMO WSN with perfect CSI can be expressed as

$$T(\mathbf{Z}) = \ln\left[\frac{p(\mathbf{Z}|\mathcal{H}_1)}{p(\mathbf{Z}|\mathcal{H}_0)}\right] = \ln\left[\prod_{k=1}^{K}\frac{p(\mathbf{z}_k|\mathcal{H}_1)}{p(\mathbf{z}_k|\mathcal{H}_0)}\right]. \tag{10}$$

The test in (10) above reduces to the form in (11) upon further simplification. Substituting the local sensor performance metrics from (1) and employing the conditional PDFs of $\mathbf{z}_k$ corresponding to $\mathbf{x}_k \in \{\mathbf{u}_k, -\mathbf{u}_k\}$, given as, $\phi(\mathbf{z}_k|\mathbf{x}_k{=}\mathbf{u}_k) \sim \mathcal{CN}(\sqrt{p_u}M\beta_k\mathbf{u}_k, M\beta_k\sigma_n^2\mathbf{I})$, $\phi(\mathbf{z}_k|\mathbf{x}_k{=}-\mathbf{u}_k) \sim \mathcal{CN}(-\sqrt{p_u}M\beta_k\mathbf{u}_k, M\beta_k\sigma_n^2\mathbf{I})$, respectively, (11) reduces to

$$T(\mathbf{Z}) = \sum_{k=1}^{K} \ln\left[\frac{P_{D,k} + (1-P_{D,k})\exp\left(\frac{-4\sqrt{p_u}}{\sigma_n^2}\Re(\mathbf{z}_k^H\mathbf{u}_k)\right)}{P_{F,k} + (1-P_{F,k})\exp\left(\frac{-4\sqrt{p_u}}{\sigma_n^2}\Re(\mathbf{z}_k^H\mathbf{u}_k)\right)}\right], \tag{12}$$

where the symbol $\Re(\cdot)$ denotes the real part. The test obtained in (12) can be further simplified for low SNR scenarios, using the approximations, $e^{-v} \approx (1-v)$ and $\ln(1+v) \approx v$, for sufficiently small values of v, as

$$T(\mathbf{Z}) = \sum_{k=1}^{K} a_k\Re(\mathbf{z}_k^H\mathbf{u}_k), \tag{13}$$

$$T(\mathbf{Z}) = \sum_{k=1}^{K} \ln\left[\frac{\phi(\mathbf{z}_k|\mathbf{x}_k=\mathbf{u}_k)\Pr(\mathbf{x}_k=\mathbf{u}_k|\mathcal{H}_1)+\phi(\mathbf{z}_k|\mathbf{x}_k=-\mathbf{u}_k)\Pr(\mathbf{x}_k=-\mathbf{u}_k|\mathcal{H}_1)}{\phi(\mathbf{z}_k|\mathbf{x}_k=\mathbf{u}_k)\Pr(\mathbf{x}_k=\mathbf{u}_k|\mathcal{H}_0)+\phi(\mathbf{z}_k|\mathbf{x}_k=-\mathbf{u}_k)\Pr(\mathbf{x}_k=-\mathbf{u}_k|\mathcal{H}_0)}\right] \tag{11}$$

where the constant a_k for the kth sensor is defined as $a_k \triangleq (P_{D,k}-P_{F,k})$. The low SNR approximation based test statistic above, which is essentially a weighted matched filter, is well suited for many practical applications that employ distributed detection, due to its significantly lower complexity. One such application is spectrum sensing, wherein the operating SNRs can be as low as -25 dB. The test statistic in (13) can be further simplified for a scenario with identical local detection performances of the sensors, i.e., $P_{D,k}=P_d$ and $P_{F,k}=P_f$, $\forall k$, as

$$T_{\text{I}}(\mathbf{Z}) = \sum_{l=1}^{L}\sum_{k=1}^{K} \Re(z_k^*(l)u_k(l)). \tag{14}$$

The result below analytically characterizes the performance of the optimal fusion rule described in (13) above.

Theorem 1. *For a given threshold γ, the probabilities of detection (P_D) and false alarm (P_{FA}) pertaining to the test statistic in (13), for distributed detection at the large antenna array equipped fusion center with perfect CSI are,*

$$P_D = Q\left(\frac{\gamma-\mu_{T|\mathcal{H}_1}}{\sigma_{T|\mathcal{H}_1}}\right), \tag{15}$$

$$P_{FA} = Q\left(\frac{\gamma-\mu_{T|\mathcal{H}_0}}{\sigma_{T|\mathcal{H}_0}}\right), \tag{16}$$

where $\mu_{T|\mathcal{H}_0}, \mu_{T|\mathcal{H}_1}, \sigma^2_{T|\mathcal{H}_0}$ and $\sigma^2_{T|\mathcal{H}_1}$ represent the mean and variance of the null and alternative hypotheses, given as

$$\mu_{T|\mathcal{H}_0} = \sum_{k=1}^{K}\sqrt{p_u}a_k c_k M\beta_k\|\mathbf{u}_k\|^2, \tag{17}$$

$$\mu_{T|\mathcal{H}_1} = \sum_{k=1}^{K}\sqrt{p_u}a_k b_k M\beta_k\|\mathbf{u}_k\|^2, \tag{18}$$

$$\sigma^2_{T|\mathcal{H}_0}=\sum_{k=1}^{K}M\beta_k a_k^2\|\mathbf{u}_k\|^2\Big(p_u M\beta_k(1-c_k^2)\|\mathbf{u}_k\|^2+\frac{\sigma_n^2}{2}\Big), \tag{19}$$

$$\sigma^2_{T|\mathcal{H}_1}=\sum_{k=1}^{K}M\beta_k a_k^2\|\mathbf{u}_k\|^2\Big(p_u M\beta_k(1-b_k^2)\|\mathbf{u}_k\|^2+\frac{\sigma_n^2}{2}\Big), \tag{20}$$

with $b_k=(2P_{D,k}-1)$ and $c_k=(2P_{F,k}-1)$.

Proof. See Appendix A. □

In practical scenarios, the matrix $\mathbf{G}$ needs to be estimated at the fusion center, which leads to estimation errors and uncertainty in the CSI. The relevant model for this CSI uncertainty and the optimal decision rule considering the availability of only imperfect CSI are determined in the next section.

IV. Optimal Fusion Rules under CSI Uncertainty

Let $\hat{\mathbf{G}}$ denote the estimated CSI. The estimation error $\mathcal{E}$ in the estimated channel matrix $\hat{\mathbf{G}}$ is defined as

$$\mathcal{E} \triangleq \hat{\mathbf{G}} - \mathbf{G}, \tag{21}$$

where the kth columns of $\hat{\mathbf{G}}$ and $\mathcal{E}$ are complex Gaussian random vectors, i.e., $\hat{\mathbf{g}}_k \sim \mathcal{CN}(\mathbf{0},\tilde{\beta}_k\mathbf{I}_M)$, $\mathbf{e}_k \sim \mathcal{CN}(\mathbf{0},\gamma_{e,k}\mathbf{I}_M)$ with $\tilde{\beta}_k \triangleq \frac{p_p\beta_k^2}{\sigma_n^2+p_p\beta_k}$ and $\gamma_{e,k} \triangleq \frac{\sigma_n^2\beta_k}{\sigma_n^2+p_p\beta_k}$. The parameters $\tilde{\beta}_k$ and $\gamma_{e,k}$ of the probability density functions (PDFs) are obtained using the minimum mean-squared error (MMSE) estimate of $\mathbf{G}$ for the orthogonal pilot matrix $\mathbf{\Phi}$, i.e., $\mathbf{\Phi}^H\mathbf{\Phi}=\mathbf{I}_K$, in (4), given as [3], $\hat{\mathbf{G}}=\frac{1}{\sqrt{p_p}}\mathbf{Y}\mathbf{\Phi}^*\mathcal{D}=(\mathbf{G}+\frac{1}{\sqrt{p_p}}\mathbf{W}_p)\mathcal{D}$, where $p_p \triangleq \tau_p p_u$ is the pilot power for $\tau_p, \tau_p \geq K$, pilot symbols, and the matrices $\mathcal{D}$ and $\mathbf{W}_p$ are given as $\mathcal{D} \triangleq \left(\frac{\sigma_n^2}{p_p}\mathbf{D}^{-1}+\mathbf{I}_K\right)^{-1}$ and $\mathbf{W}_p \triangleq \mathbf{N}\mathbf{\Phi}^*$. Under favorable propagation conditions, the different column vectors of matrix $\hat{\mathbf{G}}$, corresponding to different users, are assumed to be mutually independent with elements of $\hat{\mathbf{g}}_k$ as zero-mean i.i.d. random variables, with variance $\mathbb{E}\{|\hat{g}_{mk}|^2\}=\tilde{\beta}_k$. Hence, by the law of large numbers, it follows that $(1/M)\hat{\mathbf{G}}^H\hat{\mathbf{G}} \approx \tilde{\mathbf{D}}$, for $M \gg K$, where $[\tilde{\mathbf{D}}]_{kk}=\tilde{\beta}_k$. Similar to the perfect CSI case, the transmit vectors $\mathbf{x}(l) \in \mathbb{C}^{K\times 1}, 1\leq l\leq L$ are stacked as $\mathbf{X}=[\mathbf{x}(1),\ldots,\mathbf{x}(L)] \in \mathbb{C}^{K\times L}$ corresponding to their local decisions. Hence, the received signal matrix $\mathbf{Y}=[\mathbf{y}(1),\ldots,\mathbf{y}(L)] \in \mathbb{C}^{M\times L}$ in (4), corresponding to the CSI uncertainty model described in (21), can be equivalently obtained, as

$$\mathbf{Y}=\sqrt{p_u}\hat{\mathbf{G}}\mathbf{X}-\sqrt{p_u}\mathcal{E}\mathbf{X}+\mathbf{N}=\sqrt{p_u}\hat{\mathbf{G}}\mathbf{X}+\mathbf{W}, \tag{22}$$

where each column $\mathbf{w}(l), 1\leq l\leq L$ of the noise matrix $\mathbf{W}$, follows a Gaussian distribution $\mathbf{w}(l) \sim \mathcal{CN}(\mathbf{0},\mathbf{R}_{w(l)})$ with covariance matrix $\mathbf{R}_{w(l)}=\mathbb{E}\{\mathbf{w}(l)\mathbf{w}^H(l)\}=\big(p_u\sum_{k=1}^{K}\gamma_{e,k}+\sigma_n^2\big)\mathbf{I}_M=\sigma_w^2\mathbf{I}_M$, $\hat{\mathbf{G}}$ is the MMSE estimate of $\mathbf{G}$ and $\mathcal{E}$ denotes the estimation error matrix defined in (21). In order to obtain an analytically tractable LRT, we initially perform matched filtering on the received matrix $\mathbf{Y}$, i.e. $\tilde{\mathbf{Z}}=\hat{\mathbf{G}}^H\mathbf{Y}$. Employing the properties pertaining to favorable propagation described above, the columns $\tilde{\mathbf{z}}(l) \in \mathbb{C}^{K\times 1}$ of the filter response matrix $\tilde{\mathbf{Z}}$ can be obtained as,

$$\tilde{\mathbf{z}}(l)=\sqrt{p_u}M\tilde{\mathbf{D}}\mathbf{x}(l)+\tilde{\mathbf{w}}(l), \tag{23}$$

where the noise vector $\tilde{\mathbf{w}}(l)$ is distributed as $\tilde{\mathbf{w}}(l) \sim \mathcal{CN}(\mathbf{0},M\tilde{\mathbf{D}}\sigma_w^2)$. Similar to (9), the above model for the received vector $\tilde{\mathbf{z}}_k$, corresponding to the transmit vector $\mathbf{x}_k$ of user k, with CSI uncertainty, can be expressed as $\tilde{\mathbf{z}}_k=\sqrt{p_u}M\tilde{\beta}_k\mathbf{x}_k+\tilde{\mathbf{w}}_k$. The elements of the noise vector $\tilde{\mathbf{w}}_k$ are distributed as, $\tilde{w}_k(l) \sim \mathcal{CN}(0,M\tilde{\beta}_k\sigma_w^2)$. Therefore, the CSI uncertainty based robust test statistic $T_{\text{R}}(\tilde{\mathbf{Z}})$, for distributed detection in the massive MIMO WSN, follows from the Neyman Pearson (NP) criterion as

$$T_{\text{R}}(\tilde{\mathbf{Z}})=\ln\left[\frac{p(\tilde{\mathbf{Z}}|\mathcal{H}_1;\hat{\mathbf{G}})}{p(\tilde{\mathbf{Z}}|\mathcal{H}_0;\hat{\mathbf{G}})}\right]=\ln\left[\prod_{k=1}^{K}\frac{p(\tilde{\mathbf{z}}_k|\mathcal{H}_1;\hat{\mathbf{g}}_k)}{p(\tilde{\mathbf{z}}_k|\mathcal{H}_0;\hat{\mathbf{g}}_k)}\right]. \tag{24}$$

Consider the signaling scenario wherein the kth sensor transmits the L symbol vector $\mathbf{u}_k$ or $-\mathbf{u}_k$, to indicate the presence or absence of the target of interest, respectively. The test statistic in (24) can be further expanded as (25), with the relevant

$$T_{\mathrm{R}}(\tilde{\mathbf{Z}}) = \sum_{k=1}^{K} \ln\left[\frac{\phi(\tilde{\mathbf{z}}_k|\mathbf{x}_k=\mathbf{u}_k;\hat{\mathbf{g}}_k)\Pr(\mathbf{x}_k=\mathbf{u}_k|\mathcal{H}_1)+\phi(\tilde{\mathbf{z}}_k|\mathbf{x}_k=-\mathbf{u}_k;\hat{\mathbf{g}}_k)\Pr(\mathbf{x}_k=-\mathbf{u}_k|\mathcal{H}_1)}{\phi(\tilde{\mathbf{z}}_k|\mathbf{x}_k=\mathbf{u}_k;\hat{\mathbf{g}}_k)\Pr(\mathbf{x}_k=\mathbf{u}_k|\mathcal{H}_0)+\phi(\tilde{\mathbf{z}}_k|\mathbf{x}_k=-\mathbf{u}_k;\hat{\mathbf{g}}_k)\Pr(\mathbf{x}_k=-\mathbf{u}_k|\mathcal{H}_0)}\right] \tag{25}$$

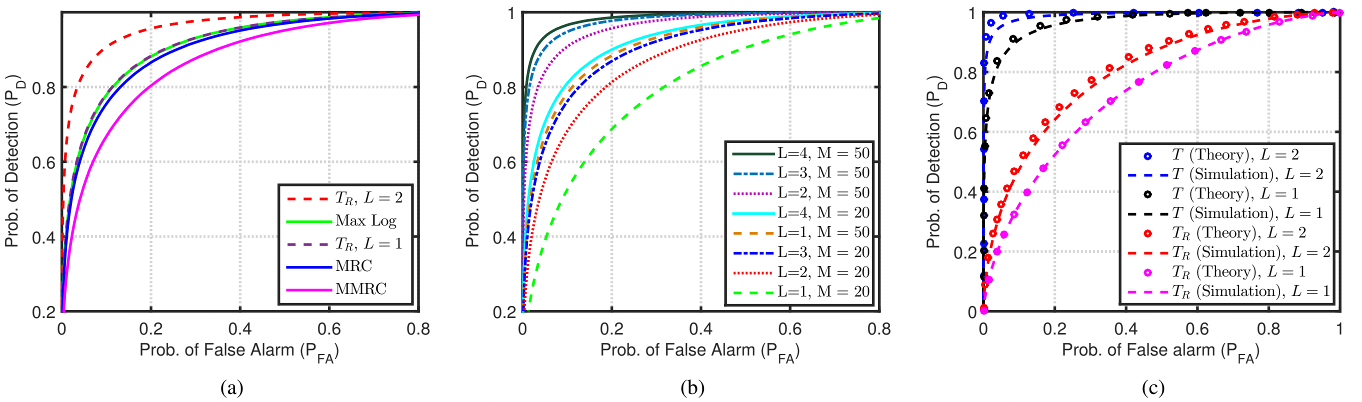

Fig. 1. Receiver operating characteristic (ROC) plot for comparing (a) Max-Log and MMRC in [10], MRC with the proposed robust detector (27) for $M = 50$ antennas, $K = 10$ sensors, $L \in \{1, 2\}$ and at SNR $p_u = -18$ dB. (b) proposed robust test in (27) with $M \in \{20, 50\}$ antennas, $K = 10$ sensors, $L \in \{1, 2, 3, 4\}$ and at SNR $p_u = -18$ dB. (c) theoretical and simulation performance of the detectors under perfect CSI in (13) and imperfect CSI in (27) with $K = 10$ sensors, $L \in \{1, 2\}$ and SNR $p_u = -20$ dB.

distributions determined as $\phi(\tilde{\mathbf{z}}_k|\mathbf{x}_k = \mathbf{u}_k; \hat{\mathbf{g}}_k) \sim \mathcal{CN}(\sqrt{p_u}M \tilde{\beta}_k\mathbf{u}_k, M\tilde{\beta}_k\sigma_w^2\mathbf{I}_L)$, $\phi(\tilde{\mathbf{z}}_k|\mathbf{x}_k{=}{-}\mathbf{u}_k; \hat{\mathbf{g}}_k) \sim \mathcal{CN}(-\sqrt{p_u}M\tilde{\beta}_k \mathbf{u}_k, M\tilde{\beta}_k\sigma_w^2\mathbf{I}_L)$. Substituting the corresponding PDFs in (25) leads to the simplified test statistic below for this scenario

$$T_{\mathrm{R}}(\tilde{\mathbf{Z}}){=}\sum_{k=1}^{K}\ln\left[\frac{P_{D,k}{+}(1{-}P_{D,k})\exp\left(\frac{-4\sqrt{p_u}}{\sigma_w^2}\Re(\tilde{\mathbf{z}}_k^H\mathbf{u}_k)\right)}{P_{F,k}{+}(1{-}P_{F,k})\exp\left(\frac{-4\sqrt{p_u}}{\sigma_w^2}\Re(\tilde{\mathbf{z}}_k^H\mathbf{u}_k)\right)}\right]. \tag{26}$$

Employing the low SNR approximation described in section III, the robust test for distributed detection reduces to

$$T_{\mathrm{R}}(\tilde{\mathbf{Z}}) = \sum_{k=1}^{K} a_k\Re(\tilde{\mathbf{z}}_k^H\mathbf{u}_k). \tag{27}$$

Finally, for the scenario with identical local sensor performance metrics, i.e., $P_{D,k} = P_d$ and $P_{F,k} = P_f$, the test statistic above further simplifies as $T_{\text{R-I}}(\tilde{\mathbf{Z}}) = \sum_{l=1}^{L}\sum_{k=1}^{K}\Re(\tilde{z}_k^*(l)u_k(l))$. Result below characterizes the detection performance for the test derived above in (27).

Theorem 2. *The performance of the robust detector in* (27) *can be characterized in terms of the probabilities of detection* (P_D) *and false alarm* (P_{FA})*, for a given threshold* γ*, as*

$$P_D = Q\left(\frac{\gamma - \mu_{T_R|\mathcal{H}_1}}{\sigma_{T_R|\mathcal{H}_1}}\right), \tag{28}$$

$$P_{FA} = Q\left(\frac{\gamma - \mu_{T_R|\mathcal{H}_0}}{\sigma_{T_R|\mathcal{H}_0}}\right), \tag{29}$$

where $\mu_{T_R|\mathcal{H}_0}, \mu_{T_R|\mathcal{H}_1}, \sigma^2_{T_R|\mathcal{H}_0}$ *and* $\sigma^2_{T_R|\mathcal{H}_1}$ *represent the mean and variance of the test statistic under the null and alternative hypotheses, given as*

$$\mu_{T_R|\mathcal{H}_0} = \sum_{k=1}^{K}\sqrt{p_u}a_kc_kM\tilde{\beta}_k\|\mathbf{u}_k\|^2, \tag{30}$$

$$\mu_{T_R|\mathcal{H}_1} = \sum_{k=1}^{K}\sqrt{p_u}a_kb_kM\tilde{\beta}_k\|\mathbf{u}_k\|^2, \tag{31}$$

$$\sigma^2_{T_R|\mathcal{H}_0}{=}\sum_{k=1}^{K}M\tilde{\beta}_ka_k^2\|\mathbf{u}_k\|^2\left(p_uM\tilde{\beta}_k(1{-}c_k^2)\|\mathbf{u}_k\|^2{+}\frac{\sigma_w^2}{2}\right),$$

$$\sigma^2_{T_R|\mathcal{H}_1}{=}\sum_{k=1}^{K}M\tilde{\beta}_ka_k^2\|\mathbf{u}_k\|^2\left(p_uM\tilde{\beta}_k(1{-}b_k^2)\|\mathbf{u}_k\|^2{+}\frac{\sigma_w^2}{2}\right),$$

with $b_k = (2P_{D,k} - 1)$ *and* $c_k = (2P_{F,k} - 1)$.

Proof. Follows along the lines similar to the proof of Theorem 1 given in Appendix A. ☐

V. SIMULATION RESULTS

This section presents simulation results to demonstrate the performance of the proposed detectors (13) and (27) and also compare their performance against the Max-Log, MMRC techniques proposed in [10] and the standard maximal ratio combiner (MRC) for the massive MIMO WSN in the existing literature. The composite transmit matrix $[\mathbf{X}]_{K\times L}$ corresponding to the local decisions of the K users is set as $\mathbf{X} \in \{+\mathbf{U}, -\mathbf{U}\}$. The sensors in the WSN are considered to be placed uniformly around the fusion center in an annular region with minimum and maximum radii $r_c = 100$ m and $r_m = 1000$ m, respectively. The large-scale fading is modeled as $\beta_k = v_k/(r_k/r_c)^2$ similar to [3], where v_k follows a log-normal distribution, i.e., $10\log_{10}v_k \sim \mathcal{N}(\mu_v, \sigma_v^2)$, where r_k denotes the distance between the kth sensor and the fusion center. The parameters $\mu_v = 4$ dB and standard deviation $\sigma_v = 2$ dB [10]. A total of $K = 10$ sensors are deployed with each having local probabilities of detection $P_{D,k}$ and false alarm $P_{F,k}$ evenly spaced in the intervals $[0.95, 0.55]$ and $[0.01, 0.1]$, respectively. We assume that $\tau_p = K$ symbols, i.e., number of sensors, are used for channel estimation, since this is the minimum required [3].

Fig. 1a plots the probability of detection (P_D) versus the probability of false alarm (P_{FA}) of the proposed scheme for $L \in \{1, 2\}$. It can be observed that the proposed schemes have an improved performance in comparison to the existing schemes presented in [10]. The proposed robust detector for $L = 1$ has a performance similar to that of the Max-Log detector. Fig. 1b shows the performance of the proposed scheme for various values of the decision vector length $L \in \{1, 2, 3, 4\}$. It can be observed that the detection performance improves with an increase in the decision vector length. Also, the performance significantly improves with the increase in the number of antennas at the fusion center, validating the benefit of employing massive MIMO technology. Fig. 1c compares the probabilities of detection and false alarm with the corresponding analytical values obtained from the results in Theorem 1 and Theorem 2, which can be seen to be in close agreement. Finally, Fig. 2 plots the probability of detection (P_D) versus number of antennas M at the fusion center for a fixed $P_{FA} = 0.01$. The trend shows that an increasing number of antennas leads to a significant improvement in the detection performance.

VI. Conclusion

This paper presented the NP criterion based optimal detection rules for distributed detection in a massive MIMO-based WSN. Analysis was carried out for both perfect and imperfect CSI scenarios incorporating also the probability of error of the local sensor decisions. Further, the system performance was characterized through the evaluation of closed form expressions for the probabilities of detection P_D and false alarm P_{FA}. Simulation results demonstrate that the proposed detectors have a better performance as compared to the Max-Log, MRC and MMRC detectors. In the future, the framework developed can facilitate capacity, CSI averaged large antenna array analyses, and also determination of the optimal transmit vectors [9] to further enhance detection performance.

Appendix A
Proof of Theorem 1

The mean (17) of the test statistic in (13) for $\mathcal{H}_0$ given as $\mu_{T|\mathcal{H}_0} = \sum_{k=1}^{K} T_{k|0}$ where $T_{k|0}$ is shown below

$$\begin{aligned} T_{k|0} =& a_k \Re(\mathbb{E}\{(\mathbf{z}_k^H \mathbf{u}_k|\mathcal{H}_0\}) \\ =& a_k \Re\big(\sqrt{p_u} M \beta_k \mathbb{E}\{\mathbf{x}_k^H|\mathcal{H}_0\}\mathbf{u}_k\big) \\ =& a_k \Re\big(\sqrt{p_u} M \beta_k (\mathbf{u}_k^H \Pr(\mathbf{x}_k = \mathbf{u}_k|\mathcal{H}_0) \\ & - \mathbf{u}_k^H \Pr(\mathbf{x}_k = -\mathbf{u}_k|\mathcal{H}_0))\mathbf{u}_k\big) \\ =& a_k \Re\big(\sqrt{p_u} M \beta_k (\mathbf{u}_k^H P_{F,k} - \mathbf{u}_k^H (1 - P_{F,k}))\mathbf{u}_k\big) \\ =& \sqrt{p_u} a_k c_k M \beta_k \|\mathbf{u}_k\|^2. \end{aligned} \tag{32}$$

Similarly, the mean of the test statistic for the alternative hypothesis $\mathcal{H}_1$ is $\mu_{T|\mathcal{H}_1} = \sum_{k=1}^{K} T_{k|1}$, where $T_{k|1}$ can be determined as shown below

$$\begin{aligned} T_{k|1} =& a_k \Re(\mathbb{E}\{(\mathbf{z}_k^H \mathbf{u}_k)|\mathcal{H}_1\}) = a_k \Re(\sqrt{p_u} M \beta_k \mathbb{E}\{\mathbf{x}_k^H|\mathcal{H}_1\}\mathbf{u}_k) \\ =& a_k \Re(\sqrt{p_u} M \beta_k \mathbf{u}_k^H (\Pr(\mathbf{x}_k = \mathbf{u}_k|\mathcal{H}_1) - \Pr(\mathbf{x}_k = -\mathbf{u}_k|\mathcal{H}_1))\mathbf{u}_k) \\ =& \sqrt{p_u} a_k M \beta_k \Re(\mathbf{u}_k^H (2P_{D,k} - 1))\mathbf{u}_k) = \sqrt{p_u} a_k b_k M \beta_k \|\mathbf{u}_k\|^2. \end{aligned} \tag{33}$$

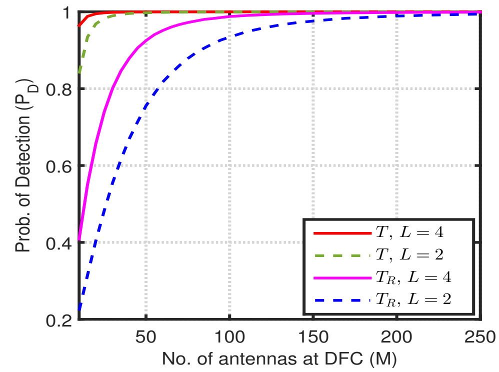

Fig. 2. P_D vs. M for perfect and imperfect CSI scenarios, for a WSN with $K = 10$ sensors, $L \in \{2, 4\}$ and SNR $p_u = -18$ dB.

The variance $\sigma^2_{T|\mathcal{H}_0}$ for the null hypothesis $\mathcal{H}_0$ is given as

$$\sigma^2_{T|\mathcal{H}_0} = \mathbb{E}\{T^2(\mathbf{Z})|\mathcal{H}_0\} - (\mathbb{E}\{T(\mathbf{Z})|\mathcal{H}_0\})^2, \tag{34}$$

where the first term in (34) is $\mathbb{E}\{T^2(\mathbf{Z})|\mathcal{H}_0\} = \mathbb{E}\{[\sum_{k=1}^{K} a_k \Re\{\mathbf{z}_k^H \mathbf{u}_k\}]^2|\mathcal{H}_0\}$ and is simplified to

$$\sum_{k=1}^{K} \Big(p_u M^2 \beta_k^2 a_k^2 ||\mathbf{u}_k||^4 + M \beta_k a_k^2 ||\mathbf{u}_k||^2 \frac{\sigma_n^2}{2}\Big). \tag{35}$$

Using (35), together with the expression in (17), yields the expression for the variance under $\mathcal{H}_0$ given in (19). Similarly, the variance $\sigma^2_{T|\mathcal{H}_1}$, corresponding to hypothesis $\mathcal{H}_1$, is obtained as given in (20).

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